Neutrino oscillations can be explained by exchange with the cosmic neutrino background

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Abstract

In order to explain neutrino oscillations, it is usually assumed that neutrinos have tiny masses, and that the mass eigenstates are superpositions of flavor eigenstates. However, the nature of neutrino mass and the mechanism by which flavors mix have not been well explained. In this article, we show that the nature of neutrino mass is a potential energy inversely proportional to the neutrino energy, which origins from the exchange of experimental neutrinos with cosmic neutrino background. The essence of neutrino flavor transition in neutrino oscillations is the exchange between neutrinos of different flavors. With the assistance of data obtained from neutrino oscillation experiments, the potential energy matrix is determined to contain only one independent parameter. The elements of the potential energy matrix have characteristics consistent with the distribution of background neutrinos predicted by the standard cosmological model. Our model can not only explain the normal neutrino oscillations, but also provide a unified perspective to explain the long-standing anomalies in some neutrino oscillation experiments. Further experimental study of these anomalies will provide a feasible way to test our model.

Introduction

It has been experimentally demonstrated more than twenty years that neutrinos can change their flavor periodically with distance over energy while they propagate, which is known as neutrino oscillations[1-5]. In order to explain this amazing phenomenon, it is usually assumed that neutrinos have tiny masses, and that the mass eigenstates are superpositions of flavor eigenstates[6,7]. In quantum mechanics, the waves associated with neutrinos of different mass have different frequencies. This gives rise to time-dependent quantum-mechanical interference effects, and the composition of the beam will change along its path. The existence of neutrino oscillations challenges two assumptions in the Standard Model of Particle Physics (SM): neutrinos have zero rest mass and the flavor of leptons is conserved[8,9]. In general, the mechanism of neutrino mass would require the addition of particle content to the SM that has never been observed, while the interpretation of leptonic flavor mixing usually requires the introduction of special flavor symmetry groups[10-12].

Despite strenuous efforts, a comprehensive understanding of the origin of neutrino mass and flavor mixing remains elusive. It is worth noting that, there is still no direct evidence to date for neutrino mass[13], and no other physical process violating lepton flavor conservation has been identified[14-16]. Furthermore, considering the long-standing anomalies observed in certain neutrino oscillation experiments that still lack satisfactory explanations[17-21], it seems that we need to revisit

our understanding of neutrino oscillations. In this regard, a theoretical model in condensed matter physics can give us some inspiration. In some magnetic materials, a moving electron can exchange with other electrons it encounters, which is used to explain the ferromagnetism exhibited by these materials (Itinerant Electron Model)[22]. Although Coulomb forces do not exist between neutrinos as they do between electrons, the exchange between neutrinos itself can explain the neutrino oscillations, as I will show below.

The explanation is based on a fundamental quantum mechanical principle - the exchange effect between identical particles. It should be emphasized that the exchange effect described here is not the neutrino-neutrino scattering mediated by the electroweak interactions. Even in the absence of interaction transmitted by vector bosons, neutrinos still adhere to certain statistical laws (neutrinos are fermions and follow Fermi-Dirac statistics). This fact indicates that when the wave functions of two neutrinos overlap in space, there must be an exchange effect between them. The situation here is analogous to the exchange of electrons between two hydrogen atoms in close proximity to each other when forming a hydrogen molecule. The Standard Cosmological Model (SCM) predicts that， the Cosmic Neutrino Background (CνB), a relic from the early university when it was about one second old, consists today of about 112 cm^{-3} neutrinos plus antineutrinos per flavor[23-25]. It is the largest neutrino density at Earth[26]. If neutrinos were massless, the CνB would be blackbody radiation at $T = 1.95$ K = 0.168 meV, corresponding to a de Broglie wavelength of about 6 millimeters. Any neutrino detected in the neutrino oscillation experiments, which we might call experimental neutrino, actually travels through the background neutrino sea before being detected. The exchange of experimental neutrinos with background neutrinos is the essence of neutrino oscillations (Fig. 1).

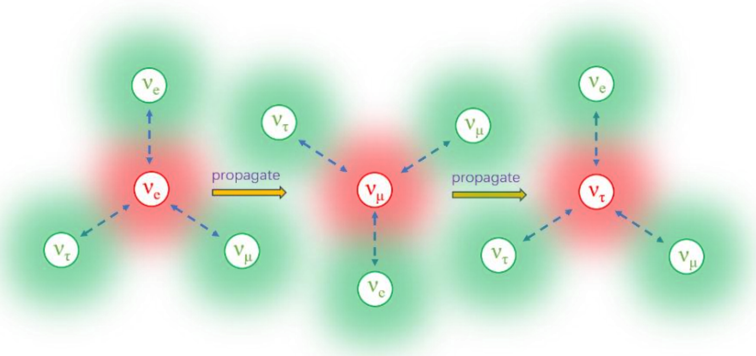

Figure 1. Schematic of neutrino oscillations. When an experimental neutrino (the red one) travels through the background neutrino sea, it can exchange with the background neutrinos (the green ones), which leads to the neutrino oscillations.

Exchange effect and potential energy matrix

Consider an experimental neutrino ν_e of energy E_1 exchanging with a background neutrino $\underline{\nu}_e$ of energy E_2. We denote the background neutrino by $\underline{\nu}_\alpha$ (α = e, μ, τ), which is used only for the sake of narrative convenience, and does not imply any essential difference between the background neutrinos and the usual experimental neutrinos. For the experimental neutrino ν_e, it can be in either the E_1 or E_2 state due to the exchanged with the background neutrino $\underline{\nu}_e$. This can be viewed as a two-state system. Assuming that the exchange amplitude is A, the corresponding Hamiltonian is

$$\hat{\mathrm{H}}=\begin{pmatrix} E_1 & A \\ A & E_2 \end{pmatrix}. \tag{1}$$

Its two eigenvalues can be derived as (See Part A in SI for details)

$$E'_1 = \left(E_1 + E_2 + \sqrt{(E_1 - E_2)^2 + 4A^2}\right)/2, \tag{2a}$$

$$E'_2 = \left(E_1 + E_2 - \sqrt{(E_1 - E_2)^2 + 4A^2}\right)/2. \tag{2b}$$

Assuming $(E_1 - E_2)^2 \gg A^2$ and $E_1 \gg E_2$, we have

$$E'_1 \approx E_1 + A^2/E_1, \tag{3a}$$

$$E'_2 \approx E_2 - A^2/E_1. \tag{3b}$$

According to Eq. (3a), the neutrino ν_e with energy E_1 acquires a potential energy inversely proportional to its energy by exchange with the background neutrino $\underline{\nu}_e$. In explaining the neutrino oscillations, it is usually assumed that the neutrino has a tiny mass m, so that the time-dependent phase factor of the wave function can be approximated to (in natural units $\hbar = c = 1$)

$$\exp(iEt) = \exp(i\sqrt{p^2 + m^2}t) \approx \exp[i(p + m^2/(2p))t], \tag{4}$$

where p is the magnitude of the neutrino momentum, and $p \gg m$ is assumed. For the neutrino with zero rest mass, Eq. (3a) can be rewritten as $E'_1 \approx p + A^2/p$. Compared to the phase factor in Eq. (4), it can be seen that in terms of the effect on the phase factor of the wave function (which is the key to explaining the neutrino oscillations), the fact that the neutrino has a tiny potential energy inversely proportional to its energy ($V = A^2/E_1$) is equivalent to giving the neutrino a tiny apparent mass m, as long as $m = \sqrt{2}|A|$ is taken.

As a neutrino ν_e travels through space, it can exchange with more than one $\underline{\nu}_e$ at the same time. We write down the total amplitude as $A_{e\underline{e}}$ and the corresponding potential energy as $V_{e\underline{e}}$. We further assume that the neutrino ν_e can also exchange with $\underline{\nu}_\mu$ and $\underline{\nu}_\tau$ in the CνB. The flavor of the leptons may be a property similar to spin polarization, and just as photons with different polarizations are still identical particles, neutrinos with different flavors can be identical particles in some sense, and thus have exchange effects as well. In contrast, charged leptons of different flavors are not identical particles because they have different rest masses. The exchange of ν_e with $\underline{\nu}_\mu$ or $\underline{\nu}_\tau$ has two effects: one is to give the neutrino the corresponding potential energy $V_{e\underline{\mu}}$ or $V_{e\underline{\tau}}$, and the other is to change the flavor of the neutrino. Once the flavor is changed, e.g., when ν_e change into ν_μ (ν_τ), the ν_μ (ν_τ) can also be exchanged with $\underline{\nu}_e$, $\underline{\nu}_\mu$, and $\underline{\nu}_\tau$ to obtain the corresponding potential energies $V_{\mu\underline{e}}$ ($V_{\tau\underline{e}}$), $V_{\mu\underline{\mu}}$ ($V_{\tau\underline{\mu}}$), and $V_{\mu\underline{\tau}}$ ($V_{\tau\underline{\tau}}$). Phenomenologically, the neutrino ν_α transitions between different flavors as it propagates and has a potential energy corresponding to each transition process. This makes the potential eigenstates of experimental neutrinos different from the flavor eigenstates. Considering that for flavor conversion, the exchange between ν_α and $\underline{\nu}_\beta$ ($\alpha, \beta = e, \mu, \tau$; $\alpha \neq \beta$) and the exchange between ν_β and $\underline{\nu}_\alpha$ are two opposite processes, i.e., the former transforms ν_α into ν_β while the latter transforms ν_β into ν_α in terms of the outcome, we can write the neutrino's potential energy matrix in flavor picture as follow

$$\hat{V} = \begin{pmatrix} V_{e\underline{e}} & V_{e\underline{\mu}} - V_{\mu\underline{e}} & V_{e\underline{\tau}} - V_{\tau\underline{e}} \\ V_{e\underline{\mu}} - V_{\mu\underline{e}} & V_{\mu\underline{\mu}} & V_{\mu\underline{\tau}} - V_{\tau\underline{\mu}} \\ V_{e\underline{\tau}} - V_{\tau\underline{e}} & V_{\mu\underline{\tau}} - V_{\tau\underline{\mu}} & V_{\tau\underline{\tau}} \end{pmatrix} = \frac{1}{E}\begin{pmatrix} A^2_{e\underline{e}} & A^2_{e\underline{\mu}} - A^2_{\mu\underline{e}} & A^2_{e\underline{\tau}} - A^2_{\tau\underline{e}} \\ A^2_{e\underline{\mu}} - A^2_{\mu\underline{e}} & A^2_{\mu\underline{\mu}} & A^2_{\mu\underline{\tau}} - A^2_{\tau\underline{\mu}} \\ A^2_{e\underline{\tau}} - A^2_{\tau\underline{e}} & A^2_{\mu\underline{\tau}} - A^2_{\tau\underline{\mu}} & A^2_{\tau\underline{\tau}} \end{pmatrix}, \tag{5}$$

where E is the energy of the experimental neutrino and $A_{\alpha\underline{\beta}}$ is the exchange amplitude corresponding

to $V_{\alpha\beta}$. According to the SCM, the three background neutrinos have similar distributions, with $\underline{\nu}_\mu$ and $\underline{\nu}_\tau$ having nearly the same distribution and $\underline{\nu}_e$ having a slightly different distribution[23-25]. We expect such a characterization to be reflected in the matrix elements of $\hat{V}$. In addition, we expect $A^2_{\alpha\underline{\alpha}}$ and $A^2_{\beta\underline{\alpha}}$ ($\alpha \neq \beta$) to be slightly different. The exchange of ν_β and $\underline{\nu}_\alpha$ ($\alpha \neq \beta$) would add an extra $\underline{\nu}_\beta$ to the background neutrinos, and the process is subject to the restriction of the Pauli exclusion principle, whereas there is no such restriction when ν_α and $\underline{\nu}_\alpha$ are exchanged.

From the above analysis, the eigenvalues of the matrix $\hat{V}$ determine the three apparent masses of the neutrinos, while its eigenstates determine how the neutrino flavor is mixed. Define the matrix $\hat{A}^2$ as

$$\hat{A}^2 = \begin{pmatrix} A^2_{ee} & A^2_{e\mu} & A^2_{e\tau} \\ A^2_{\mu e} & A^2_{\mu\mu} & A^2_{\mu\tau} \\ A^2_{\tau e} & A^2_{\tau\mu} & A^2_{\tau\tau} \end{pmatrix}, \tag{6}$$

where, $A^2_{\alpha\alpha} = A^2_{\alpha\underline{\alpha}}$ (α = e, μ, τ), $A^2_{e\mu} = A^2_{\mu e} = A^2_{e\underline{\mu}} - A^2_{\mu\underline{e}}$, $A^2_{\mu\tau} = A^2_{\tau\mu} = A^2_{\mu\underline{\tau}} - A^2_{\tau\underline{\mu}}$, and $A^2_{e\tau} = A^2_{\tau e} = A^2_{e\underline{\tau}} - A^2_{\tau\underline{e}}$. From Eqs. (5) and (6), we have $\hat{V} = \hat{A}^2/E$, and thus there is a relation $V_i = A_i^2/E$ between the eigenvalues V_i of $\hat{V}$ and the eigenvalues A_i^2 of $\hat{A}^2$. The three eigenvalues of the matrix $\hat{A}^2$ can be solved as

$$A_1^2 = \overline{A^2} + 2|S|^{1/3}\cos[(\theta - 4\pi)/3], \tag{7a}$$
$$A_2^2 = \overline{A^2} + 2|S|^{1/3}\cos[(\theta - 2\pi)/3], \tag{7b}$$
$$A_3^2 = \overline{A^2} + 2|S|^{1/3}\cos(\theta/3), \tag{7c}$$

where, $|S|$ and θ ($0 \leq \theta \leq \pi$) are functions of the matrix elements of matrix $\hat{A}^2$, and $\overline{A^2} \equiv (A^2_{ee} + A^2_{\mu\mu} + A^2_{\tau\tau})/3$ (See Part B in SI for details).

According to the previous analysis, the three eigenvalues A_i^2 ($i = 1, 2, 3$) correspond to the three apparent masses M_i ($i = 1, 2, 3$) of neutrinos, and satisfy $M_i^2 = 2A_i^2$. From Eqs. (7a-c) we have

$$M_3^2 - M_1^2 = 2(A_3^2 - A_1^2) = 4\sqrt{3}|S|^{1/3}\sin(2\pi/3 - \theta/3)) > 0, \tag{8a}$$
$$M_3^2 - M_2^2 = 2(A_3^2 - A_2^2) = 4\sqrt{3}|S|^{1/3}\sin(\pi/3 - \theta/3) > 0, \tag{8b}$$
$$M_2^2 - M_1^2 = 2(A_2^2 - A_1^2) = 4\sqrt{3}|S|^{1/3}\sin(\theta/3) > 0. \tag{8c}$$

Current neutrino oscillation experiments have not been able to fully determine the ordering of the three neutrino masses m_i (i = 1, 2, 3). There are two possibilities, which are referred to as normal ordering (NO, $m_3 > m_2 > m_1$) and inverse ordering (IO, $m_2 > m_1 > m_3$), respectively. From Eqs. (8a-c), we have $m_1 = M_1$, $m_2 = M_2$, $m_3 = M_3$ for NO, and $m_1 = M_2$, $m_2 = M_3$, $m_3 = M_1$ for IO. Either in the case of NO or IO, using the available data from neutrino oscillation experiments, we can determine the values of $|S|^{1/3}$ and θ. Taking the case of NO as an example, using Eqs. (8a) and (8c), and taking the best-fit values[27] $\Delta^2_{21} \equiv m_2^2 - m_1^2 = 7.42 \times 10^{-5} eV^2$ and $\Delta^2_{31} = m_3^2 - m_1^2 = 2.514 \times 10^{-3} eV^2$, we obtain

$$\sin(\theta/3)/\sin[(2\pi - \theta)/3] = (m_2^2 - m_1^2)/(m_3^2 - m_1^2) \approx 0.0295. \tag{9}$$

Solving the above equation gives $\theta \approx 4.46^\circ$. Substituting it into Eq. (8a) gives $|S|^{1/3} \approx 4.130 \times 10^{-4}\ eV^2$. From Eqs. (7a-c), we have

$$A_1^2 = \overline{A^2} - 4.314 \times 10^{-4} eV^2, \tag{10a}$$
$$A_2^2 = \overline{A^2} - 3.943 \times 10^{-4} eV^2, \tag{10b}$$
$$A_3^2 = \overline{A^2} + 8.257 \times 10^{-4} eV^2. \tag{10c}$$

To explain the neutrino oscillations, it is usually assumed that the flavor eigenstates $|\nu_\alpha\rangle$ $(\alpha = e, \mu, \tau)$ of neutrinos are a superposition of three mass eigenstates $|\nu_i\rangle$ $(i = 1, 2, 3)$, i.e.,

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i}|\nu_i\rangle. \tag{11}$$

The $U_{\alpha i}$ are elements of the unitary matrix $\hat{U}$, which is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix to be parameterized in terms of three mixing angles (θ_{12}, θ_{23}, θ_{13}), and one phase parameter δ_{CP},

$$\hat{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}, \tag{12}$$

where, $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$. From the above analysis, $m_i^2 = 2A_i^2$ (in the case of NO), so the eigenstate of the apparent mass m_i is the eigenstate of A_i^2. This means that in the flavor picture, the eigenstate corresponding to the eigenvalue A_i^2 is the i-th column of the matrix $\hat{U}$. We can diagonalize the matrix $\hat{A}^2$ (Eq. (6)) with matrix $\hat{U}$, that is,

$$\hat{U}^\dagger \hat{A}^2 \hat{U} = diag\ (A^2), \tag{13}$$

where the diagonal matrix element in row i of the diagonal matrix $diag(A^2)$ is A_i^2. The Eq. (13) can be equivalently written as

$$\hat{A}^2 = \hat{U} diag\ (A^2) \hat{U}^\dagger. \tag{14}$$

From Eqs. (12) and (14), the elements of matrix $\hat{A}^2$ can be solved as

$$A_{ee}^2 = \overline{A^2} + c_{12}^2 c_{13}^2 \Delta A_1^2 + s_{12}^2 c_{13}^2 \Delta A_2^2 + s_{13}^2 \Delta A_3^2, \tag{15a}$$

$$\begin{aligned} A_{e\mu}^2 = &(-c_{12}s_{12}c_{13}c_{23} - c_{12}^2 c_{13}s_{13}s_{23}e^{-i\delta_{CP}})\Delta A_1^2 \\ &+(c_{12}s_{12}c_{13}c_{23} - s_{12}^2 c_{13}s_{13}s_{23}e^{-i\delta_{CP}})\Delta A_2^2 \\ &+c_{13}s_{13}s_{23}e^{-i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{15b}$$

$$\begin{aligned} A_{e\tau}^2 = &(c_{12}s_{12}c_{13}s_{23} - c_{12}^2 c_{13}s_{13}c_{23}e^{-i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}s_{12}c_{13}s_{23} - s_{12}^2 c_{13}s_{13}c_{23}e^{-i\delta_{CP}})\Delta A_2^2 \\ &+c_{13}s_{13}c_{23}e^{-i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{15c}$$

$$\begin{aligned} A_{\mu e}^2 = &(-c_{12}s_{12}c_{13}c_{23} - c_{12}^2 c_{13}s_{13}s_{23}e^{i\delta_{CP}})\Delta A_1^2 \\ &+(c_{12}s_{12}c_{13}c_{23} - s_{12}^2 c_{13}s_{13}s_{23}e^{i\delta_{CP}})\Delta A_2^2 \\ &+c_{13}s_{13}s_{23}e^{i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{15d}$$

$$\begin{aligned} A_{\mu\mu}^2 = &\overline{A^2} + (s_{12}^2 c_{23}^2 + c_{12}^2 s_{13}^2 s_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_1^2 \\ &+(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_2^2 \\ &+c_{13}^2 s_{23}^2 \Delta A_3^2, \end{aligned} \tag{15e}$$

$$\begin{aligned} A_{\mu\tau}^2 = &(-s_{12}^2 c_{23}s_{23} + c_{12}^2 s_{13}^2 c_{23}s_{23} + c_{12}s_{12}s_{13}c_{23}^2 e^{-i\delta_{CP}} - c_{12}s_{12}s_{13}s_{23}^2 e^{i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}^2 c_{23}s_{23} + s_{12}^2 s_{13}^2 c_{23}s_{23} - c_{12}s_{12}s_{13}c_{23}^2 e^{-i\delta_{CP}} + c_{12}s_{12}s_{13}s_{23}^2 e^{i\delta_{CP}})\Delta A_2^2 \\ &+c_{13}^2 c_{23}s_{23}\Delta A_3^2, \end{aligned} \tag{15f}$$

$$\begin{aligned} A_{\tau e}^2 = &(c_{12}s_{12}c_{13}s_{23} - c_{12}^2 c_{13}s_{13}c_{23}e^{i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}s_{12}c_{13}s_{23} - s_{12}^2 c_{13}s_{13}c_{23}e^{i\delta_{CP}})\Delta A_2^2 \\ &+c_{13}s_{13}c_{23}e^{i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{15g}$$

$$\begin{aligned} A_{\tau\mu}^2 = &(-s_{12}^2 c_{23}s_{23} + c_{12}^2 s_{13}^2 c_{23}s_{23} + c_{12}s_{12}s_{13}c_{23}^2 e^{i\delta_{CP}} - c_{12}s_{12}s_{13}s_{23}^2 e^{-i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}^2 c_{23}s_{23} + s_{12}^2 s_{13}^2 c_{23}s_{23} - c_{12}s_{12}s_{13}c_{23}^2 e^{i\delta_{CP}} + c_{12}s_{12}s_{13}s_{23}^2 e^{-i\delta_{CP}})\Delta A_2^2 \\ &+c_{13}^2 c_{23}s_{23}\Delta A_3^2, \end{aligned} \tag{15h}$$

$$\begin{aligned} A_{\tau\tau}^2 = &\overline{A^2} + (s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_1^2 \\ &+(c_{12}^2 s_{23}^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_2^2 \\ &+c_{13}^2 c_{23}^2 \Delta A_3^2, \end{aligned} \tag{15i}$$

where $\Delta A_1^2 = 2|S|^{1/3}\cos((\theta - 4\pi)/3)$, $\Delta A_2^2 = 2|S|^{1/3}\cos((\theta - 2\pi)/3)$, and $\Delta A_3^2 = 2|S|^{1/3}\cos(\theta/3)$.

In the case of NO, the best-fit value of δ_{CP} is 195º, and here we may wish to take δ_{CP} = 180º to simplify the analysis and to be consistent with the earlier taking of the matrix $\hat{A}^2$ (Eq. (6)) to be a

real matrix. In principle, for any δ_{CP}, we can calculate the matrix $\hat{A}^2$ with the help of Eqs. (15a-i). However, if the matrix elements of $\hat{U}$ are imaginary, then the non-diagonal matrix elements of $\hat{A}^2$ will also be imaginary. This means that the matrix $\hat{V}$ (Eq. (5)) needs to be introduced with the corresponding phase factors in its non-diagonal matrix elements. For the three mixing angles, we take the best-fit values[27] $\theta_{12} = 33.44^{\circ}$, $\theta_{23} = 49.0^{\circ}$ and $\theta_{13} = 8.57^{\circ}$. Substituting the above taken values and Eqs. (10a-c) into Eqs. (15a-i), with $10^{-4}\ eV^2$ as the unit of the matrix elements, we have

$$\hat{A}^2 = \begin{pmatrix} \overline{A^2} - 3.925 & -1.275 & -1.332 \\ -1.275 & \overline{A^2} + 2.825 & 5.956 \\ -1.332 & 5.956 & \overline{A^2} + 1.100 \end{pmatrix}. \tag{16}$$

The parameters ($\Delta_{ij}^2 (\equiv m_i^2 - m_j^2)$, θ_{ij}, δ_{CP}) measured by the neutrino oscillation experiments have completely determined the values of $A_{\alpha\beta}^2 (\alpha \neq \beta)$ and $A_{\alpha\alpha}^2 - A_{\beta\beta}^2$ (Eqs. (15a-i)). This can also be seen directly from Eq. (16). Also, it can be seen that $A_{\alpha\beta}^2 (\alpha \neq \beta)$ and $A_{\alpha\alpha}^2 - A_{\beta\beta}^2$ are in the same order of magnitude. This is consistent with our expectations. It can also be seen that between the diagonal elements A_{ee}^2, $A_{\mu\mu}^2$ and $A_{\tau\tau}^2$, the values of $A_{\mu\mu}^2$ and $A_{\tau\tau}^2$ are somewhat closer. All these features are still present when taking $\delta_{CP} = 0$ and even in the case of IO ($\delta_{CP} = 0, \pi$) (**Table 1**, see Part C in SI for details).

Mass Order	δ_{CP}	A_{ee}^2	$A_{\mu\mu}^2$	$A_{\tau\tau}^2$	$A_{e\mu}^2$	$A_{e\tau}^2$	$A_{\mu\tau}^2$
NO	π	$\overline{A^2} - 3.925$	$\overline{A^2} + 2.825$	$\overline{A^2} + 1.110$	-1.275	-1.332	5.956
NO	0	$\overline{A^2} - 3.925$	$\overline{A^2} + 2.775$	$\overline{A^2} + 1.496$	1.496	1.077	5.963
IO	π	$\overline{A^2} + 3.868$	$\overline{A^2} - 2.841$	$\overline{A^2} - 1.028$	1.524	1.088	-6.164
IO	0	$\overline{A^2} + 3.868$	$\overline{A^2} - 2.891$	$\overline{A^2} - 0.977$	-1.304	-1.344	-6.157

Table 1. The calculated values of matrix elements of $\hat{A}^2$ (in the unit of $10^{-4} eV^2$).

The experimental parameters Δ_{ij}^2 and the mixing matrix $\hat{U}$ (in terms of θ_{ij}) of neutrino oscillations can be derived from the matrix $\hat{A}^2$ (Eq. (16)), which suggests that these parameters are in fact determined by $A_{\alpha\beta}^2 (\alpha \neq \beta)$ and $A_{\alpha\alpha}^2 - A_{\beta\beta}^2$, i.e., that the neutrino oscillations are mainly determined by the absolute differences between the three background neutrino distributions. The relative differences between $A_{\alpha\alpha}^2 (\alpha = e,\ \mu,\ \tau)$ (i.e., $\left|A_{\alpha\alpha}^2 - A_{\beta\beta}^2\right| / \overline{A^2}$) depend on the value of $\overline{A^2}$. If $\overline{A^2} = 500$ is taken, the relative difference between $A_{\alpha\alpha}^2 (\alpha = e,\ \mu,\ \tau)$ is on the order of one percent by Eq. (16); if $\overline{A^2} = 5000$ is taken, the difference is on the order of one thousandth. From the above analysis, the study of neutrino oscillations provides information on the differences in the distribution of background neutrinos with different flavors.

Understanding the anomalies in neutrino oscillations

Given $\overline{A^2}$ in Eq. (16), the matrix $\hat{A}^2$ is uniquely determined. Although the available experimental data on neutrino oscillations does not determine the value of $\overline{A^2}$, the anomalies in some neutrino oscillation experiments provide some hints as to the range of possible values of $\overline{A^2}$. There are four long-standing anomalies in the short-baseline neutrino experiments[17-21,28,29]. Two arise from the apparent oscillatory appearance of electron (anti)neutrinos in relatively pure muon-(anti)neutrino beams. The other two anomalies are associated with an overall normalization discrepancy of electron antineutrinos expected both from conventional fission reactors (the Reactor Antineutrino Anomaly) and in the radioactive decay of ^{71}Ga (the Gallium Anomaly). In these latter cases, no oscillatory signature is observed, but the overall normalization deficit can be ascribed to rapid oscillations at a high $\Delta_{ij}^2 \sim 1\ eV^2$ that are overaged out and appear as an overall deficit. The neutrino experiments in which the above anomalies occur share several common features: firstly,

they all involve high-flux neutrino sources. For example, in the case of the Ga anomaly [19], the neutrino flux Φ_ν near the neutrino source is on the order of $10^{14}\ cm^{-2}s^{-1}$; secondly, there are obvious tensions in the range of possible values of Δ_{ij}^2 (and θ_{ij}) obtained from different experiments; thirdly, a clear oscillatory signal dependent on *L/E* (*L* is the detector-to-neutrino-source distance, and *E* is the neutrino energy) has not been observed in almost all the cases (except for the Neutrino-4 experiment[30]). In the following we will show that our model provides a unified perspective for understanding the neutrino anomalies mentioned above.

In the previous analysis, we implicitly assumed that the CvB is not perturbed by exchanges with experimental neutrinos, and thus the matrix $\hat{A}^2$ is definite. This may not be true for the short-baseline neutrino experiments described above. The SCM predicts that the flux of the cosmic neutrino background is $\Phi_{\underline{\nu}} \sim 10^{12}\ cm^{-2}s^{-1}$. If the neutrino flux Φ_ν near the neutrino source used in a neutrino oscillation experiment satisfies $\Phi_\nu \gg \Phi_{\underline{\nu}}$, then the CvB near the neutrino source may be disturbed by exchanges with experimental neutrinos. This will change the values of $A_{\alpha\beta}^2(\alpha \neq \beta)$ and $A_{\alpha\alpha}^2 - A_{\beta\beta}^2$. As a result, the mixing matrix $\widehat{U}$ and the apparent mass m_i of the neutrinos are changed. At a given perturbation, Δ_{ij}^2 may increase, and this will lead to more rapid oscillations. The result would be a further enhancement of the perturbation to the background neutrinos. This nonlinear effect may allow Δ_{ij}^2 to increase rapidly to the order of $\overline{A^2}$. We can make a rough estimate. In unit of $10^{-4}\ eV^2$, taking $A_{ee}^2 = 6000$, $A_{\mu\mu}^2 = 5000$ and $A_{\tau\tau}^2 = 4000$ (i.e., a 20% perturbation with respect to the mean value $\overline{A^2} = 5000$, specifically, 20% of $\underline{\nu}_\tau$ transitions to $\underline{\nu}_e$) and simply taking $A_{\alpha\beta}^2 = A_{\alpha\alpha}^2 - A_{\beta\beta}^2$, i.e., $A_{e\mu}^2 = 1000$, $A_{\mu\tau}^2 = 1000$, and $A_{e\tau}^2 = 2000$. Direct calculations give the three eigenvalues of $A_3^2 \approx 7895$, $A_2^2 \approx 4397$, and $A_1^2 \approx 2708$, respectively. (It is worth pointing out that, for experimental neutrinos with energy *E* of about 1 MeV^2 or greater, the potential energy ($V_i = A_i^2\ /\ E$) corresponding to the above values is quite small.) We have $\Delta_{31}^2 = 2(A_3^2 - A_1^2) \approx 10357$. This means that if $\overline{A^2} = 5000$, a 20% perturbation could lead to $\Delta_{ij}^2 \sim 1\ eV^2$. Because the details of the neutrino sources are different in various high-flux neutrino experiments, the perturbation of the CvB is also different. This explains why there is a tension between the fitted data from different experiments. In addition, the neutrino flux varies with distance, and thus the perturbation of the CvB also varies with distance, which causes Δ_{ij}^2 to vary with distance. It is anticipated that the oscillatory signal dependent on L/E will be eliminated, so that the experimental results reflect only some kind of averaging effect. From the above analysis, the study of anomalies in neutrino oscillation experiments will provide information on the absolute magnitude of the background neutrino flux. Such studies also provide a feasible way to test our theory. If the correlation between neutrino flux and the anomalies is systematically studied by experiments and a clear relationship between them is confirmed, this will be a strong support for our theory.

The SCM predicts that neutrinos and antineutrinos in the cosmic neutrino background have the same distribution, which means that the results of the above analysis apply to both neutrinos and antineutrinos. On the other hand, if one analyzes the oscillation data for neutrinos and antineutrinos separately and confirms that Δ_{ij}^2 and θ_{ij} have different best-fit values in the two cases, then according to our model, this would suggest that neutrinos and antineutrinos in the CvB have different distributions. As a final note, since the zero-mass neutrinos and antineutrinos possess opposite helicity, an exchange between them would violate the conservation of angular momentum and thus need not be considered.

Conclusions

In summary, we put forward a new interpretation for the neutrino oscillations. According to our

interpretation, the nature of the neutrino mass is a potential energy inversely proportional to the energy of the neutrino, which is acquired by exchange with CνB. The observed change in neutrino flavors in oscillation experiments results from the exchange between experimental neutrinos and background neutrinos with different flavors. We have analyzed the potential energy matrix of the experimental neutrinos using the parameters measured by the neutrino oscillation experiments, and the obtained results have features consistent with the distribution of the CνB predicted by the SCM. Moreover, according to our interpretation, if the neutrino experiments involve a neutrino flux much larger than that of CνB, the neutrino exchange will likely perturb the distribution of CνB, causing the apparent mass of the experimental neutrino to change. This provides a unified new perspective for explaining the anomalies in various neutrino experiments which usually involve high-flux neutrino sources.

Unlike many existing neutrino theories that are difficult to test experimentally (e.g., the Seesaw Model[31]), our model can be confirmed or falsified by feasible experiments. For instance, studying the relationship between the anomalies mentioned above and the magnitude of the neutrino flux would provide a feasible way to verify our model. Furthermore, our model suggests that neutrino oscillations do not imply that neutrinos have mass or that the conservation of leptonic flavor is violated. If future cosmological observations[32] show that the upper limit on the sum of neutrino masses ($m_1 + m_2 + m_3$) is less than 50 meV, this would disprove the popular interpretation of neutrino oscillations, and thus be a piece of evidence in favor of our model. On the other hand, if experiments confirm the existence of neutrinoless ββ decays or other lepton flavor violating decays, that would be a piece of evidence against our model.

From the theoretical point of view, it would also be a strong support for our model if one could calculate the matrix (Eq. (16)) starting from the basic principles of quantum mechanics and the distribution function of background neutrinos predicted by the SCM. Further research is needed on related issues. In addition to the cosmic neutrino background, the entire universe is permeated by the cosmic microwave background. Just as experimental neutrinos are exchanged with background neutrinos, photons from distant galaxies are expected to be exchanged with the cosmic microwave background as it travels through cosmic space. The effect of this exchange is an interesting subject for study.

Data availability

The datasets used and analysed during the current study available from the corresponding author on reasonable request.

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Acknowledgements

The work was funded through the Natural Science Foundation of China (no. 11504321).

Author contributions

Shen, X. S. contributes all to the whole work.

Competing interests

The author declares no competing interests.

Supplementary Information (SI)

Neutrino oscillations can be explained by exchange with the cosmic neutrino background

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Part A. Find the eigenvalues of matrix $\hat{\mathrm{H}}$ （Eq. (1) in the main text）:

$$\hat{\mathrm{H}}=\begin{pmatrix} E_1 & A \\ A & E_2 \end{pmatrix}.$$

Solution:

Denoting the eigenvalues as $E_i'(i=1,2)$, we have

$$\det\begin{pmatrix} E_1-E_i' & A \\ A & E_2-E_i' \end{pmatrix}=0. \qquad \text{(A1)}$$

That is

$$(E_i')^2-(E_1+E_2)E_i'+E_1E_2-A^2=0. \qquad \text{(A2)}$$

The solutions are

$$E_{1,2}'=[E_1+E_2\pm\sqrt{(E_1-E_2)^2+4A^2}\,]/2. \qquad \text{(A3)}$$

If $(E_1-E_2)^2 \gg A^2$ and $E_1 \gg E_2$, we have

$$E_{1,2}'\approx[E_1+E_2\pm(E_1-E_2)(1+2A^2/(E_1-E_2)^2)]/2 \qquad \text{(A4)}$$

namely,

$$E_1'\approx E_1+A^2/(E_1-E_2)\approx E_1+A^2/E_1, \qquad \text{(A5)}$$

$$E_2'\approx E_2-A^2/(E_1-E_2)\approx E_2-A^2/E_1. \qquad \text{(A6)}$$

Part B. Find the eigenvalues of matrix $\hat{A}^2$ (Eq. (6) in the main text):

$$\hat{A}^2 = \begin{pmatrix} A^2_{ee} & A^2_{e\mu} & A^2_{e\tau} \\ A^2_{\mu e} & A^2_{\mu\mu} & A^2_{\mu\tau} \\ A^2_{\tau e} & A^2_{\tau\mu} & A^2_{\tau\tau} \end{pmatrix},$$

where, $A^2_{e\mu} = A^2_{\mu e}, A^2_{\mu\tau} = A^2_{\tau\mu}$ and $A^2_{e\tau} = A^2_{\tau e}$.

Solution:

Denoting the eigenvalues as $A_i^2 (i = 1,2,3)$, we have

$$\det\begin{pmatrix} A^2_{ee} - A_i^2 & A^2_{e\mu} & A^2_{e\tau} \\ A^2_{\mu e} & A^2_{\mu\mu} - A_i^2 & A^2_{\mu\tau} \\ A^2_{\tau e} & A^2_{\tau\mu} & A^2_{\tau\tau} - A_i^2 \end{pmatrix} = 0. \tag{B1}$$

That is

$$\begin{aligned} &(A_i^2)^3 - (A^2_{ee} + A^2_{\mu\mu} + A^2_{\tau\tau})(A_i^2)^2 \\ &+(A^2_{ee}A^2_{\mu\mu} + A^2_{\mu\mu}A^2_{\tau\tau} + A^2_{ee}A^2_{\tau\tau} - (A^2_{e\mu})^2 - (A^2_{\mu\tau})^2 - (A^2_{e\tau})^2)A_i^2 \\ &+A^2_{ee}(A^2_{\mu\tau})^2 + A^2_{\mu\mu}(A^2_{e\tau})^2 + A^2_{\tau\tau}(A^2_{e\mu})^2 - A^2_{ee}A^2_{\mu\mu}A^2_{\tau\tau} - 2A^2_{e\mu}A^2_{\mu\tau}A^2_{e\tau} = 0. \end{aligned} \tag{B2}$$

Let

$$a = -(A^2_{ee} + A^2_{\mu\mu} + A^2_{\tau\tau}), \tag{B3}$$

$$b = A^2_{ee}A^2_{\mu\mu} + A^2_{\mu\mu}A^2_{\tau\tau} + A^2_{ee}A^2_{\tau\tau} - (A^2_{e\mu})^2 - (A^2_{\mu\tau})^2 - (A^2_{e\tau})^2, \tag{B4}$$

$$c = A^2_{ee}(A^2_{\mu\tau})^2 + A^2_{\mu\mu}(A^2_{e\tau})^2 + A^2_{\tau\tau}(A^2_{e\mu})^2 - A^2_{ee}A^2_{\mu\mu}A^2_{\tau\tau} - 2A^2_{e\mu}A^2_{\mu\tau}A^2_{e\tau}. \tag{B5}$$

We have

$$(A_i^2)^3 + a(A_i^2)^2 + bA_i^2 + c = 0\,. \tag{B6}$$

The solutions are

$$A_1^2 = \overline{A^2} + \omega\cdot\sqrt[3]{-g/2 + \sqrt{g^2/4 + f^3/27}} + \omega^2\cdot\sqrt[3]{-g/2 - \sqrt{g^2/4 + f^3/27}} \tag{B7}$$

$$A_2^2 = \overline{A^2} + \omega^2\cdot\sqrt[3]{-g/2 + \sqrt{g^2/4 + f^3/27}} + \omega\cdot\sqrt[3]{-g/2 - \sqrt{g^2/4 + f^3/27}} \tag{B8}$$

$$A_3^2 = \overline{A^2} + \sqrt[3]{-g/2 + \sqrt{g^2/4 + f^3/27}} + \sqrt[3]{-g/2 - \sqrt{g^2/4 + f^3/27}} \tag{B9}$$

where, $\omega = e^{i2\pi/3}$, and

$$\overline{A^2} \equiv -a/3 = (A^2_{ee} + A^2_{\mu\mu} + A^2_{\tau\tau})/3, \tag{B10}$$

$$
\begin{aligned}
& f \equiv b - a^2/3 \\
& = [A_{ee}^2 A_{\mu\mu}^2 + A_{\mu\mu}^2 A_{\tau\tau}^2 + A_{\tau\tau}^2 A_{ee}^2 - (A_{ee}^2)^2 - (A_{\mu\mu}^2)^2 - (A_{\tau\tau}^2)^2]/3 \\
& -(A_{e\mu}^2)^2 - (A_{\mu\tau}^2)^2 - (A_{e\tau}^2)^2,
\end{aligned} \tag{B11}
$$

$$
\begin{aligned}
& g \equiv c - ab/3 + 2a^3/27 \\
& = -2A_{e\mu}^2 A_{\mu\tau}^2 A_{e\tau}^2 + 2[A_{ee}^2 (A_{\mu\tau}^2)^2 + A_{\mu\mu}^2 (A_{e\tau}^2)^2 + A_{\tau\tau}^2 (A_{e\mu}^2)^2]/3 \\
& -4A_{ee}^2 A_{\mu\mu}^2 A_{\tau\tau}^2/9 - 2[(A_{ee}^2)^3 + (A_{\mu\mu}^2)^3 + (A_{\tau\tau}^2)^3]/27 \\
& -\{A_{ee}^2[(A_{e\mu}^2)^2 + (A_{e\tau}^2)^2] + A_{\mu\mu}^2[(A_{e\mu}^2)^2 + (A_{\mu\tau}^2)^2] + A_{\tau\tau}^2[(A_{\mu\tau}^2)^2 + (A_{e\tau}^2)^2]\}/3 \\
& +\{A_{ee}^2[(A_{\mu\mu}^2)^2 + (A_{\tau\tau}^2)^2] + A_{\mu\mu}^2[(A_{ee}^2)^2 + (A_{\tau\tau}^2)^2] + A_{\tau\tau}^2[(A_{ee}^2)^2 + (A_{\mu\mu}^2)^2]\}/9.
\end{aligned} \tag{B12}
$$

Since $\hat{A}^2 = (\hat{A}^2)^T$, the three eigenvalues $A_i^2 (i = 1,2,3)$ of $\hat{A}^2$ are all real. This implies that

$$g^2/4 + f^3/27 \le 0. \tag{B13}$$

Let

$$-g/2 + \sqrt{g^2/4 + f^3/27} = |S| e^{i\theta} (0 \le \theta \le \pi), \tag{B14}$$

we have

$$-g/2 - \sqrt{g^2/4 + f^3/27} = |S| e^{-i\theta}. \tag{B15}$$

From Eqs. B7-9, B14 and B15, we have

$$A_1^2 = \overline{A^2} + e^{i2\pi/3} \cdot |S|^{1/3} e^{i\theta/3} + e^{i4\pi/3} \cdot |S|^{1/3} e^{-i\theta/3}, \tag{B16}$$

$$A_2^2 = \overline{A^2} + e^{i4\pi/3} \cdot |S|^{1/3} e^{i\theta/3} + e^{i2\pi/3} \cdot |S|^{1/3} e^{-i\theta/3}, \tag{B17}$$

$$A_3^2 = \overline{A^2} + |S|^{1/3} e^{i\theta/3} + |S|^{1/3} e^{-i\theta/3}, \tag{B18}$$

which are，

$$A_1^2 = \overline{A^2} + 2|S|^{1/3} \cos((\theta - 4\pi)/3), \tag{B19}$$

$$A_2^2 = \overline{A^2} + 2|S|^{1/3} \cos((\theta - 2\pi)/3), \tag{B20}$$

$$A_3^2 = \overline{A^2} + 2|S|^{1/3} \cos(\theta/3). \tag{B21}$$

Part C. Find the matrix $\hat{A}^2$ from Eq. $\hat{A}^2 = \hat{U} diag(A^2) \hat{U}^{\dagger}$ (Eq. 14 in the main text).

Solution:

Let $\Delta A_1^2 = 2|S|^{1/3}\cos((\theta-4\pi)/3)$, $\Delta A_2^2 = 2|S|^{1/3}\cos((\theta-2\pi)/3)$, and $\Delta A_3^2 = 2|S|^{1/3}\cos(\theta/3)$, we have

$$A_1^2 = \overline{A^2} + \Delta A_1^2, \tag{C1}$$

$$A_2^2 = \overline{A^2} + \Delta A_2^2, \tag{C2}$$

$$A_3^2 = \overline{A^2} + \Delta A_3^2. \tag{C3}$$

Thus we have

$$diag(A^2) = \begin{pmatrix} \overline{A^2} + \Delta A_1^2 & 0 & 0 \\ 0 & \overline{A^2} + \Delta A_2^2 & 0 \\ 0 & 0 & \overline{A^2} + \Delta A_3^2 \end{pmatrix}. \tag{C4}$$

Because

$$\hat{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}, \tag{C5}$$

we have

$$\begin{aligned} \hat{A}^2 &\equiv \begin{pmatrix} A_{ee}^2 & A_{e\mu}^2 & A_{e\tau}^2 \\ A_{\mu e}^2 & A_{\mu\mu}^2 & A_{\mu\tau}^2 \\ A_{\tau e}^2 & A_{\tau\mu}^2 & A_{\tau\tau}^2 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \\ &\cdot \begin{pmatrix} \overline{A^2} + \Delta A_1^2 & 0 & 0 \\ 0 & \overline{A^2} + \Delta A_2^2 & 0 \\ 0 & 0 & \overline{A^2} + \Delta A_3^2 \end{pmatrix} \\ &\cdot \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta_{CP}} & s_{12}s_{23} - c_{12}s_{13}c_{23}e^{-i\delta_{CP}} \\ s_{12}c_{13} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{-i\delta_{CP}} \\ s_{13}e^{i\delta_{CP}} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix}. \end{aligned} \tag{C6}$$

The matrix elements of $\hat{A}^2$ can be solved as

$$A_{ee}^2 = \overline{A^2} + c_{12}^2 c_{13}^2 \Delta A_1^2 + s_{12}^2 c_{13}^2 \Delta A_2^2 + s_{13}^2 \Delta A_3^2, \tag{C7}$$

$$\begin{aligned} A_{e\mu}^2 &= (-c_{12}s_{12}c_{13}c_{23} - c_{12}^2 c_{13}s_{13}s_{23}e^{-i\delta_{CP}})\Delta A_1^2 \\ &+(c_{12}s_{12}c_{13}c_{23} - s_{12}^2 c_{13}s_{13}s_{23}e^{-i\delta_{CP}})\Delta A_2^2 + c_{13}s_{13}s_{23}e^{-i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{C8}$$

$$\begin{aligned} A_{e\tau}^2 &= (c_{12}s_{12}c_{13}s_{23} - c_{12}^2 c_{13}s_{13}c_{23}e^{-i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}s_{12}c_{13}s_{23} - s_{12}^2 c_{13}s_{13}c_{23}e^{-i\delta_{CP}})\Delta A_2^2 + c_{13}s_{13}c_{23}e^{-i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{C9}$$

$$\begin{aligned} A_{\mu e}^2 &= (A_{e\mu}^2)^* = (-c_{12}s_{12}c_{13}c_{23} - c_{12}^2 c_{13}s_{13}s_{23}e^{i\delta_{CP}})\Delta A_1^2 \\ &+(c_{12}s_{12}c_{13}c_{23} - s_{12}^2 c_{13}s_{13}s_{23}e^{i\delta_{CP}})\Delta A_2^2 + c_{13}s_{13}s_{23}e^{i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{C10}$$

$$\begin{aligned} A_{\mu\mu}^2 &= \overline{A^2} + (s_{12}^2 c_{23}^2 + c_{12}^2 s_{13}^2 s_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_1^2 \\ &+(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_2^2 + c_{13}^2 s_{23}^2 \Delta A_3^2, \end{aligned} \tag{C11}$$

$$\begin{aligned} A_{\mu\tau}^2 &= (-s_{12}^2 c_{23}s_{23} + c_{12}^2 s_{13}^2 c_{23}s_{23} + c_{12}s_{12}s_{13}c_{23}^2 e^{-i\delta_{CP}} - c_{12}s_{12}s_{13}s_{23}^2 e^{i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}^2 c_{23}s_{23} + s_{12}^2 s_{13}^2 c_{23}s_{23} - c_{12}s_{12}s_{13}c_{23}^2 e^{-i\delta_{CP}} + c_{12}s_{12}s_{13}s_{23}^2 e^{i\delta_{CP}})\Delta A_2^2 + c_{13}^2 c_{23}s_{23}\Delta A_3^2, \end{aligned} \tag{C12}$$

$$\begin{aligned} A_{\tau e}^2 &= (A_{e\tau}^2)^* = (c_{12}s_{12}c_{13}s_{23} - c_{12}^2 c_{13}s_{13}c_{23}e^{i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}s_{12}c_{13}s_{23} - s_{12}^2 c_{13}s_{13}c_{23}e^{i\delta_{CP}})\Delta A_2^2 + c_{13}s_{13}c_{23}e^{i\delta_{CP}}\Delta A_3^2, \end{aligned} \tag{C13}$$

$$\begin{aligned} A_{\tau\mu}^2 &= (A_{\mu\tau}^2)^* = (-s_{12}^2 c_{23}s_{23} + c_{12}^2 s_{13}^2 c_{23}s_{23} + c_{12}s_{12}s_{13}c_{23}^2 e^{i\delta_{CP}} - c_{12}s_{12}s_{13}s_{23}^2 e^{-i\delta_{CP}})\Delta A_1^2 \\ &+(-c_{12}^2 c_{23}s_{23} + s_{12}^2 s_{13}^2 c_{23}s_{23} - c_{12}s_{12}s_{13}c_{23}^2 e^{i\delta_{CP}} + c_{12}s_{12}s_{13}s_{23}^2 e^{-i\delta_{CP}})\Delta A_2^2 + c_{13}^2 c_{23}s_{23}\Delta A_3^2, \end{aligned} \tag{C14}$$

$$\begin{aligned} A_{\tau\tau}^2 &= \overline{A^2} + (s_{12}^2 s_{23}^2 + c_{12}^2 s_{13}^2 c_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_1^2 \\ &+(c_{12}^2 s_{23}^2 + s_{12}^2 s_{13}^2 c_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}s_{23}\cos\delta_{CP})\Delta A_2^2 + c_{13}^2 c_{23}^2 \Delta A_3^2. \end{aligned} \tag{C15}$$

From Eqs. C7, C11 and C15, it can be seen that the value of $A_{\alpha\alpha}^2 - A_{\beta\beta}^2$ does not depend on $\overline{A^2}$.

In the case of NO（$m_3 > m_2 > m_1$）, from Eqs. 10a-c in the main text, we have

$$\Delta A_1^2 = -4.314\times10^{-4} eV^2, \tag{C16}$$

$$\Delta A_2^2 = -3.943\times10^{-4} eV^2, \tag{C17}$$

$$\Delta A_1^2 = 8.257\times10^{-4} eV^2. \tag{C18}$$

In the case of IO（$m_2 > m_1 > m_3$）, by the analysis in the main text, we have

$$m_1^2/2 = \overline{A^2} + 2|S|^{1/3}\cos((\theta-2\pi)/3), \tag{C19}$$

$$m_2^2/2 = \overline{A^2} + 2|S|^{1/3}\cos(\theta/3), \tag{C20}$$

$$m_3^2/2=\overline{A^2}+2|S|^{1/3}\cos((\theta-4\pi)/3)\,. \tag{C21}$$

Thus we have

$$\begin{aligned}m_2^2-m_1^2&=2(A_3^2-A_2^2)=4|S|^{1/3}[\cos(\theta/3)-\cos((2\pi-\theta)/3)]\\&=4\sqrt{3}|S|^{1/3}\sin((\pi-\theta)/3),\end{aligned} \tag{C22}$$

$$\begin{aligned}m_2^2-m_3^2&=2(A_3^2-A_1^2)=4|S|^{1/3}[\cos(\theta/3)-\cos((4\pi-\theta)/3)]\\&=4\sqrt{3}|S|^{1/3}\sin((2\pi-\theta)/3).\end{aligned} \tag{C23}$$

Taking the best-fit values $m_2^2-m_1^2=7.42\times10^{-5}eV^2$ and $m_2^2-m_3^2=2.571\times10^{-3}eV^2$, we have

$$(m_2^2-m_1^2)/(m_2^2-m_3^2)=\sin((\pi-\theta)/3)/\sin((2\pi-\theta)/3)\approx0.0289. \tag{C24}$$

From the above equation, we have

$$\theta=175.64°, \tag{C25}$$

and thus we have

$$|S|^{1/3}=4.225\times10^{-4}eV^2. \tag{C26}$$

In the case of IO, we have

$$\Delta A_1^2=m_1^2/2-\overline{A^2}=2|S|^{1/3}\cos((\theta-2\pi)/3)=4.034\times10^{-4}eV^2, \tag{C27}$$

$$\Delta A_2^2=m_2^2/2-\overline{A^2}=2|S|^{1/3}\cos(\theta/3)=4.409\times10^{-4}eV^2, \tag{C28}$$

$$\Delta A_3^2=m_3^2/2-\overline{A^2}=2|S|^{1/3}\cos((\theta-4\pi)/3)=-8.447\times10^{-4}eV^2. \tag{C29}$$

In addition, in the case of NO, the best-fit values $\theta_{12}=33.44°,\theta_{23}=49.0°,\theta_{13}=8.57°$ are taken; in the case of IO, the best-fit values $\theta_{12}=33.45°,\theta_{23}=49.3°,\theta_{13}=8.61°$ are taken. Substituting the above values into the matrix elements of $\hat{A}^2$, the results are shown in **Table 1** in the main text.